Exercise 47

Use logarithmic differentiation to find the derivative of the function.

$$y = (\cos x)^x$$

Solution

Take the natural logarithm of both sides and use the properties of logarithms to simplify the right side.

$$\ln y = \ln(\cos x)^x$$

$$= x \ln \cos x$$

Differentiate both sides with respect to x.

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln \cos x)$$

$$\frac{1}{y} \cdot \frac{d}{dx}(y) = \left[\frac{d}{dx}(x)\right] \ln \cos x + x \left[\frac{d}{dx}(\ln \cos x)\right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = (1) \ln \cos x + x \left[\frac{1}{\cos x} \cdot \frac{d}{dx}(\cos x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \ln \cos x + x \left[\frac{1}{\cos x} \cdot (-\sin x)\right]$$

$$\frac{dy}{dx} = y(\ln \cos x - x \tan x)$$

$$= (\cos x)^x(\ln \cos x - x \tan x)$$